



Secrétariat

Distr.  
GÉNÉRALE

ST/SY/AC.10/C.3/2006/55  
20 avril 2006

FRANÇAIS  
Original: ANGLAIS

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COMITÉ D'EXPERTS DU TRANSPORT DES  
MARCHANDISES DANGEREUSES ET DU SYSTÈME  
GÉNÉRAL HARMONISÉ DE CLASSIFICATION ET  
D'ÉTIQUETAGE DES PRODUITS CHIMIQUES

Sous-Comité d'experts du transport  
des marchandises dangereuses

Vingt-neuvième session  
Genève, 3-12 (matin) juillet 2006  
Point 13 de l'ordre du jour provisoire

QUESTIONS DIVERSES

Propositions d'amendements au *Manuel d'épreuves et de critères*: Méthodes d'épreuve  
pour la détermination de la température de décomposition auto-accélérée (TDAA)

Communication de l'Association internationale des marchandises dangereuses  
et des conteneurs (IDGCA)

**Introduction**

1. Nombreux sont les problèmes qui ont trait à la détermination de la température de décomposition auto-accélérée (TDAA) ou point de décomposition exothermique (définition ambiguë de ce paramètre, particularités des méthodes d'épreuve recommandées dans le *Manuel d'épreuves et de critères*), notamment en ce qui concerne les matières solides. Cet état des choses exige que soient révisés de nombreux paragraphes de la section 28 du *Manuel d'épreuves et de critères*. Les propositions contenues dans le présent document sont fondées sur l'analyse comparative détaillée, présentée en annexe au présent document, des méthodes recommandées dans ledit *Manuel d'épreuves et de critères*.

## Propositions

### Sous-section 28.2 (Méthodes d'épreuve)

2. Il peut être montré que toutes les méthodes recommandées dans le *Manuel d'épreuves et de critères* ne sont pas également applicables aux liquides et aux matières solides (voir la section 2 de l'annexe au présent document). Il est donc proposé de remplacer dans l'actuel paragraphe 28.2.2 la phrase libellée comme suit:

«Chacune des méthodes énumérées est utilisable pour les matières solides, les liquides, les matières pâteuses et les dispersions.»

par la phrase, ainsi conçue:

«Les méthodes H1 et H4 sont applicables aux matières solides, aux liquides, aux matières pâteuses et aux dispersions. Les méthodes H2 et H3 sont applicables aux liquides faiblement visqueux seulement.»

### Sous-section 28.3 (Conditions d'épreuve)

3. Il est proposé de ne pas utiliser la perte de chaleur spécifique comme critère d'équivalence sur le plan thermique des emballages de dimensions différentes, mais d'introduire le temps de refroidissement en tant que critère physiquement mieux établi et plus fiable (voir l'introduction et la sous-section 2.3.4 de l'annexe). Les principaux avantages de ce critère sont les suivants:

- a) Il correspond entièrement au libellé du 28.3.5 qui stipule qu'il faut tenir compte «de la quantité de matière, des dimensions du colis, du transfert de chaleur dans la matière et du transfert de chaleur à travers l'emballage vers l'environnement»;
- b) Les expressions analytiques obtenues permettent de calculer le temps de refroidissement pour des emballages de formes diverses, si les propriétés physiques de la matière et le transfert de chaleur à travers la surface du colis sont connus;
- c) Si le temps de refroidissement a été mesuré et que l'expression analytique existante est applicable à un emballage, alors les valeurs précises du transfert de chaleur vers l'intérieur (transfert de chaleur dans la matière) et vers l'extérieur peuvent être évaluées. Ces valeurs sont nécessaires pour l'extrapolation à une échelle plus grande;
- d) Pour les liquides, l'égalité des temps de refroidissement pour les emballages de dimensions différentes est équivalente à l'égalité des pertes de chaleur spécifique;
- e) Pour les matières solides, le temps de refroidissement a une signification physique claire, tandis que le temps de demi-refroidissement n'est qu'un paramètre empirique, non défini du point de vue physique;
- f) Pour les récipients dont les formes et les dimensions ainsi que les propriétés physiques diffèrent, la concordance des temps de refroidissement assure qu'ils ont des comportements thermiques équivalents. En raison de cela, le temps de refroidissement est un point de départ sûr pour l'extrapolation à une échelle plus grande.

4. À la suite de ce remplacement, il est proposé de recommander l'étalonnage des emballages au moyen de la mesure du temps de refroidissement plutôt que du temps de demi-refroidissement. Ce paramètre a un autre avantage: les résultats de sa mesure ne dépendent pas de la position du détecteur dans le colis. À l'inverse, les résultats de la mesure du temps de demi-refroidissement dépendent de la position du détecteur, dans les cas où le transfert de chaleur varie en fonction des différentes surfaces traversées du récipient (le vase de DEWAR en est un exemple).

5. La méthode de détermination du temps de refroidissement est semblable à la méthode mentionnée au 28.3.6 (sixième ligne et lignes suivantes). La phrase existante libellée comme suit:

«Pour établir une échelle, il pourrait être nécessaire de mesurer en continu la température de la matière et du voisinage avant d'utiliser une régression linéaire pour calculer les coefficients de l'équation ci-après:

$$\ln\{T - T_a\} = c_0 + C \times t \quad (1a)$$

dans laquelle       $T$       =      température de la matière ( $^{\circ}\text{C}$ );  
 $T_a$       =      température ambiante ( $^{\circ}\text{C}$ );  
 $C_0$       =       $\ln\{\text{température initiale de la matière} - \text{température ambiante initiale}\}$ ;  
 $C$       =       $L/C_p (\text{s}^{-1})$ ;  
 $t$       =      temps (s).»

doit être remplacée par la phrase, ainsi conçue:

«Pour mesurer le temps de refroidissement, il est nécessaire de mesurer en continu la température de la matière et du voisinage, puis de choisir la partie linéaire des valeurs après une période de transition et d'utiliser une régression linéaire pour calculer les coefficients de l'équation ci-après:

$$\ln\{T - T_a\} = c_0 + \omega \times t \quad (1b)$$

dans laquelle       $T$       =      température de la matière ( $^{\circ}\text{C}$ );  
 $T_a$       =      température ambiante ( $^{\circ}\text{C}$ );  
 $C_0$       =       $\ln\{\text{température initiale de la matière} - \text{température ambiante initiale}\}$ ;  
 $\omega$       =      temps de refroidissement ( $\text{s}^{-1}$ );  
 $t$       =      temps (s).».

En comparant les équations (1a) et (1b), il ressort qu'elles sont identiques pour les liquides, impliquant que le temps de refroidissement est directement proportionnel à la perte de chaleur spécifique. En outre, pour un récipient contenant un liquide (citerne très secouée), il n'y a pas de temps de transition observable, en raison de quoi les points expérimentaux pour le refroidissement, reportés dans un diagramme d'axes  $\ln\{T - T_a\}$  – temps, sont alignés sur une droite.

6. Puisque la perte de chaleur spécifique ne peut servir comme critère pour les matières solides, il est proposé de supprimer la deuxième partie du tableau 28.3 «Pour les solides».

7. Il est proposé d'inclure un nouveau 28.3.8, qui décrive la méthode permettant de calculer les temps de refroidissement pour des emballages de formes différentes et soit ainsi conçu:

### **<28.3.8 Calculs du temps de refroidissement pour des récipients de formes différentes**

28.3.8.1 Le temps de refroidissement peut aisément être calculé pour des corps de formes différentes si les propriétés physico-thermiques de la matière et le transfert de chaleur vers l'extérieur sont connus.

#### **Équation pour la sphère:**

$$\omega = a[\mu_1(Bi)/r]^2$$

où	r	=	rayon de la sphère (m);
	Bi	=	critère de Biot, $Bi=Ur/\lambda$ ;
	a	=	diffusivité thermique ( $m^2/s$ ); $a = \lambda/c_p/\rho$ ,
	$c_p$	=	chaleur spécifique ( $J/kg/K$ );
	$\rho$	=	densité ( $kg/m^3$ );
	U	=	coefficient de transfert de chaleur ( $W/m^2/k$ );
	$\mu_1(Bi)$	=	première racine de l'équation caractéristique $\operatorname{tg}\mu = -\mu/(Bi - 1)$

Les valeurs de  $\mu_1(Bi)$  sont données dans le tableau 28.4

#### **Équation pour un cylindre (fût):**

$$\omega = a[\mu_{ls}^2 / (h/2)^2 + \mu_{lc}^2 / r^2]; \quad Bi_s = \frac{U_s h / 2}{\lambda}; \quad Bi_c = \frac{U_c r}{\lambda},$$

où	r	=	rayon (m)
	h	=	hauteur (m)
	$\mu_{lc}(Bi_c)$	=	première racine de l'équation caractéristique $\operatorname{ctg}\mu_s = \mu_s / Bi_s$
	$\mu_{ls}(Bi_s)$	=	première racine de l'équation caractéristique $\frac{J_0(\mu_c)}{J_1(\mu_c)} = \mu_c / (Bi_c - 1)$
indices	c	=	surface latérale du fût;
	s	=	surfaces au sommet et à la base du fût

Les valeurs de  $\mu_1(Bi)$  sont données dans le tableau 28.4

#### **Équation pour un parallélépipède (boîte rectangulaire):**

$$\omega = a \sum_{i=1}^3 \left( \frac{\mu_{ls}(Bi_{si})}{h_i/2} \right)^2; \quad Bi_{si} = \frac{U_{si} h_i / 2}{\lambda}$$

où  $h_i$  = dimensions de la boîte (m);  
 $\mu_{1s}(Bi_{si})$  = première racine de l'équation caractéristique  $ctg\mu_s = \mu_s / Bi_{si}$   
 $U_{si}, Bi_{si}$  = coefficient de transfert de chaleur et valeur correspondante du critère de Biot sur chacune des paires de surfaces opposées de la boîte

Les valeurs de  $\mu_1(Bi)$  sont données dans le tableau 28.4.

**Tableau 28.4: Premières racines des équations caractéristiques**

Bi	$\mu_1(Bi)$ pour la sphère	$\mu_1(Bi)$ pour le parallélépipède	$\mu_1(Bi)$ pour le cylindre
0,02	0,24450	0,141	0,1995
0,04	0,34500	0,1987	0,2814
0,06	0,42170	0,2425	0,3438
0,08	0,48600	0,2791	0,396
0,1	0,54230	0,3111	0,4417
0,2	0,75930	0,4328	0,617
0,4	1,05280	0,5932	0,8516
0,6	1,26140	0,7051	1,0184
0,8	1,43200	0,791	1,149
1	1,57080	0,8603	1,2558
1,5	1,83660	0,9882	1,4569
2	2,02880	1,0769	1,5994
3	2,28890	1,1925	1,7887
4	2,45570	1,2646	1,9081
5	2,57040	1,3138	1,9898
6	2,65370	1,3496	2,049
7	2,71650	1,3766	2,0937
8	2,76540	1,3978	2,1286
9	2,80440	1,4149	2,1566
10	2,83630	1,4289	2,1795
15	2,93200	1,4729	2,2509
20	2,98750	1,4961	2,288
30	3,03700	1,5202	2,3261

Bi	$\mu_1(Bi)$ pour la sphère	$\mu_1(Bi)$ pour le parallélépipède	$\mu_1(Bi)$ pour le cylindre
50	3,07880	1,54	2,3572
Approximation polynomiale de la dépendance $\mu_1(Bi)$ : $\mu_1 = a_0 + \sum_1^4 a_i X^i$ ; $X = \sqrt{Bi}$			
$a_0$	- 0,0736	- 0,0288	- 0,0529
$a_1$	2,1513	1,2059	1,7423
$a_2$	0,5748	- 0,3724	- 0,501
$a_3$	0,0689	0,0518	0,0651
$a_4$	-0,0031	-0,0027	-0,0031
$R^2$	0,9997	0,9998	0,9998

L'approximation polynomiale donne une précision suffisante (les coefficients de corrélation pour toutes les formes sont très proches de un) et peut être employée pour les calculs.».

#### Sous-section 28.4 (Série H: Dispositions d'épreuve)

##### 28.4.2 «Épreuve H.2: Épreuve de stockage adiabatique»

8. La méthode de détermination de la TDAA, prévue par l'épreuve H.2, repose principalement sur la théorie de Semenov qui est fondée sur le modèle d'un récipient très secoué. L'épreuve H.2 ne peut donc s'appliquer qu'à des liquides faiblement visqueux. En outre, les évaluations de la TDAA obtenues au moyen de cette épreuve H.2 sont valables pour les réactions en une étape sans auto-accélération (telles que les réactions autocatalytiques ou les réactions en chaîne) (voir la section 2.2 de l'annexe). Il est donc proposé de remplacer la dernière phrase du paragraphe 28.4.2.1.1, libellée comme suit:

«Cette méthode convient pour n'importe quel type d'emballage, y compris les GRV et les citernes.»

par la phrase, ainsi conçue:

«Cette méthode convient pour n'importe quel type d'emballage, y compris les GRV et les citernes contenant des matières liquides qui se décomposent par une réaction en une étape sans auto-accélération.».

##### 28.4.3 «Épreuve H.3: Épreuve de stockage isotherme»

9. Comme pour l'épreuve H.2, la méthode de détermination de la TDAA prévue par l'épreuve H.3 repose principalement sur la théorie de Semenov qui est fondée sur le modèle d'un récipient très secoué. L'épreuve H.3 ne peut donc s'appliquer qu'à des liquides faiblement visqueux. En outre, les évaluations de la TDAA obtenues au moyen de l'épreuve H.3 sont valables pour les réactions en une étape (voir la section 2.2 de l'annexe au présent document).

Il est donc proposé de remplacer l'avant-dernière phrase du paragraphe 28.4.3.1.1, libellée comme suit:

«Cette méthode convient pour n'importe quel type d'emballage, y compris les GRV et les citerne.»

par la phrase, ainsi conçue:

«Cette méthode convient pour tout type d'emballage, y compris les GRV et les citerne contenant des matières liquides qui se décomposent par une réaction en une étape sans auto-accélération et/ou par une réaction autocatalytique.».

#### **28.4.4 «Épreuve H.4: Épreuve de stockage avec accumulation de chaleur»**

10. Il est stipulé au 28.4.4.1.1 que «La méthode repose sur la théorie de l'explosion sous l'effet de la chaleur de Semenov, c'est-à-dire que l'on considère que la résistance principale aux flux thermiques se trouve aux parois du récipient.». Cela veut dire que l'épreuve H.4 tout entière s'applique à la détermination de la TDAA pour les liquides. Pour les matières solides, la procédure d'extrapolation à une échelle plus grande décrite dans le *Manuel d'épreuves et de critères* ne permet pas de prévoir correctement la TDAA (voir la section 2.3 de l'annexe). Il est donc proposé de mettre explicitement l'accent sur cette réserve en remplaçant la dernière phrase du 28.4.4.1.1, libellée comme suit:

«La méthode peut être utilisée pour déterminer la TDAA d'une matière dans son emballage, y compris les GRV et les petites citerne (jusqu'à 2 m<sup>3</sup>).»

par la phrase, ainsi conçue:

«La méthode peut être utilisée pour déterminer la TDAA d'une matière liquide dans son emballage, y compris les GRV et les petites citerne (jusqu'à 2 m<sup>3</sup>).»

Elle peut être utilisée pour déterminer la TDAA d'une matière solide dans son emballage, pour des volumes jusqu'à 0,03 m<sup>3</sup>, sous réserve d'appliquer une procédure spéciale d'extrapolation à une échelle plus grande (voir le 28.4.4.2.9).».

11. Il est en outre proposé d'ajouter un nouveau 28.4.4.2.9 où sont décrites des procédures appropriées d'extrapolation à une échelle plus grande, qui permettent une prévision fiable de la TDAA pour des colis contenant des matières solides. Un texte approprié pour le paragraphe proposé peut être élaboré sur la base de l'analyse présentée dans la section 2.3 de l'annexe au présent document.

Trois méthodes d'extrapolation à une échelle plus grande, décrites dans la section susmentionnée, peuvent en particulier être recommandées:

- a) Une nouvelle méthode fondée sur la théorie du mode de refroidissement régulier;
- b) La méthode de Bowes qui est une version améliorée de la méthode de Grewer;

c) Une méthode fondée sur l'égalité des temps de demi-refroidissement, mesurés pour chacun des vases de Dewar et chaque colis particulier.

## **Propositions d'ordre général**

### ***Unification des définitions de la TDAA***

12. Deux différentes définitions de la TDAA sont citées dans le *Manuel d'épreuves et de critères*. La première se réfère à l'épreuve TDAA des États-Unis (l'épreuve H1) et à l'épreuve de stockage avec accumulation de chaleur (épreuve H4):

**La TDAA est la température ambiante (dans l'étuve) la plus basse à laquelle la température de surchauffe à l'intérieur d'un emballage commercial donné dépasse la température de l'étuve de 6 °C, après une période de sept jours (168 heures) ou moins.**

La deuxième définition correspond à l'épreuve de stockage adiabatique (épreuve H2) et à l'épreuve de stockage isotherme (épreuve H3):

**La TDAA est la température ambiante arrondie au multiple de 5 °C le plus proche.**

13. La première définition est axée sur deux paramètres importants: la température de surchauffe maximale admissible et la période minimale d'induction acceptable. La deuxième ne propose qu'un paramètre: la température critique de l'explosion thermique arrondie au multiple de 5 °C le plus proche. Aucune limite n'est fixée pour la période d'induction. Ce désaccord peut conduire à des évaluations très différentes de la TDAA en raison de motifs suivants:

a) Il peut être démontré que, si la TDAA est prise au sens de la première définition, la corrélation entre la TDAA et la température critique dépend des propriétés de la réaction. Plus particulièrement, si une réaction sans auto-accélération se produit dans une matière, la TDAA est légèrement plus basse que la température critique et est atteinte après une période inférieure à sept jours. Dans le cas d'une réaction autocatalytique, la TDAA est toujours supérieure à la température critique et cette différence peut atteindre 5 à 15 °C (voir les sections 2.1 et 2.2 de l'annexe au présent document);

b) En raison du désaccord entre les définitions, les TDAA déterminées au moyen des épreuve H1 ou H4 pour la matière se décomposant par une réaction avec auto-accélération peuvent être très différentes l'une de l'autre.

14. Il est donc proposé d'élaborer une définition unifiée qui couvrirait toutes les épreuves.

### ***Adjonction d'une nouvelle méthode de détermination de la TDAA***

15. Toutes les méthodes expérimentales recommandées par le *Manuel d'épreuves et de critères* pour la détermination de la TDAA sont accompagnées de réserves importantes, en particulier lorsqu'il s'agit de matières solides. En outre, plusieurs cas concrets ne sont pas couverts par les méthodes existantes. Ce sont les cas suivants:

- a) Détermination de la TDAA pour des citernes à fort tonnage (camions-citernes, wagons-citernes) ou des piles de colis;
- b) Évaluation des marges de sécurité lors du transport de chargements en vrac de produits auto-réactifs (par exemple, le transport d'engrais à base de nitrate d'ammonium);
- c) Évaluation des dangers potentiels lors du transport ou du stockage pendant plus de sept jours des produits auto-réactifs.

16. Tous ces problèmes peuvent être résolus en appliquant une méthode de simulation fondée sur la cinétique. La modélisation de l'emballage vers l'explosion thermique dans les matières solides est une tâche complexe des points de vue physico-chimique et mathématique. Néanmoins, le fait de disposer d'ordinateurs personnels modernes très performants équipés de logiciels appropriés permet d'appliquer cette méthode perfectionnée à grande échelle. En combinant les avantages des méthodes expérimentales et des méthodes de simulation pour la détermination de la TDAA, on peut obtenir des résultats beaucoup plus fiables et solides pour une gamme plus étendue de problèmes concrets (voir la section 3 de l'annexe pour de plus amples détails). On peut, à l'aide de cette méthode, jeter les bases d'une définition unifiée généralisée de la TDAA. Une formulation possible est donnée dans la section 3 de l'annexe.

17. En tenant compte de ce qui précède, il est proposé d'inclure la méthode de simulation fondée sur la cinétique dans la liste des méthodes recommandées par le *Manuel d'épreuves et de critères*.

### **Incidences sur la sécurité**

18. L'application de méthodes plus fiables pour la détermination de la TDAA permet de renforcer la sécurité au cours du transport et du stockage des matières auto-réactives.

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**Annexe (ANGLAIS SEULEMENT)**

(Text reproduced as submitted)

Comparative Analysis of the Methods for SADT Determination.

### 1. Introduction

The self-accelerating decomposition temperature (the SADT) is an important parameter that characterizes thermal hazard under transport conditions of condensed self-reactive substances. The SADT has been introduced into the international practice by the United Nations “*Recommendations on the Transport of Dangerous Goods, Manual of Tests and Criteria*” (TDG) [1]. The Globally Harmonized System (GHS) [2] had inherited the SADT as a classification criterion for self-reactive substances. According to TDG the SADT is defined as “the lowest temperature at which self-accelerating decomposition may occur with a substance in the packaging as used in transport”. Important feature of the SADT is that it is not an intrinsic property of a substance but “...a measure of the combined effect of the ambient temperature, decomposition kinetics, packaging size and the heat transfer properties of the substance and its packaging” [1].

If the SADT  $\leq 50^{\circ}\text{C}$  for organic peroxides and  $\leq 55^{\circ}\text{C}$  for self-reactive substances, the following control and emergency temperatures are set for a packaging (Table 1).

Table 1 : Derivation of control and emergency temperatures

Receptacle	Group	SADT	Control t-re	Emergency t-re
Single packagings and IBSs	1	20°C or less	20°C below SADT	10°C below SADT
	2	Over 20°C to 35°C	15°C below SADT	10°C below SADT
	3	Over 35°C	10°C below SADT	5°C below SADT
Portable tanks	4	<50°C	10°C below SADT	5°C below SADT

The Manual recommends four tests for determining the SADT:

1. The United States SADT test (US SADT test) H1;
2. Adiabatic storage test (AST) H2;
3. Isothermal storage test (IST) H3;
4. Heat accumulation storage test (Dewar test) H4.

The H1 test foresees the experimental determination of the SADT for a commercial packaging. The H4 test is also based on experimental determination of the SADT for a small Dewar vessel which is supposed to be representative for a commercial packaging provided that the special scale-up procedure is used.

The H2 and H3 tests are based on the use of adiabatic and isothermal calorimetric technique respectively with the following estimation of the SADT.

The US SADT test is the only method that gives the direct and, hence, the most reliable answer. Nevertheless it is used rather rarely because of its expensiveness. Moreover this test can be applied only for packagings of up to 220 liters so that large tanks or intermediate bulk containers (IBCs) turn out to be out of the scope of this test. The H2-H4 tests are very attractive because they are based on the lab-scale experiments, don't involve such a large amount of reactive product and therefore are less expensive and dangerous. At the same time all these tests have essential limitations that should be taken into account when selecting one or another test. Special attention should be drawn to the fact that there exists an element of uncertainty with regard to the SADT definition.

Detailed analysis of problems related to the SADT determination methods have been presented by Fisher [3], numerous more recent papers are focused on correctness of some particular methods (see, for instance, [4-10]). This paper continues discussion of certain important aspects of the SADT determination methods. The consideration is illustrated by the abstract simulated examples that are capable of conveying the ideas without superfluous details.

## **2. Overview of the methods for SADT determination**

### **2.1 The United States SADT test H1**

The US SADT test H1 (and the Dewar test H4) is based on the following definition of the SADT:

**SADT is the lowest environment (oven) temperature at which overheating in the middle of the specific commercial packaging exceeds 6 °C after a lapse of the period of seven days (168 hours) or less** (D1)

This period is measured from the time when the packaging center temperature reaches 2 °C below the oven temperature (Fig.1a).

The US SADT test represents the series of full-scale experiments that are carried out with the specific commercial packagings of a product. The packaging is inserted in the test chamber (oven) and is maintained at a constant oven temperature. The temperature in the center of the packaging is monitored. Every experiment of the series is implemented with the new packaging. The step of the oven temperature variation is 5 °C.

The H1 test provides direct experimental determination of the SADT therefore there are not any particular problems concerning the test by itself. Nevertheless the challenge is issued by an element of uncertainty with regard to the SADT definition.

The general definition states that the SADT is “the lowest temperature at which self-accelerating decomposition may occur...” but it doesn't contain any quantitative measure that would allow to judge whether self-acceleration occurred or not. The next definition (D1) gives such a measure (overheat in the center in combination with period after which it is reached) but the physical ground of these figures remains unclear.

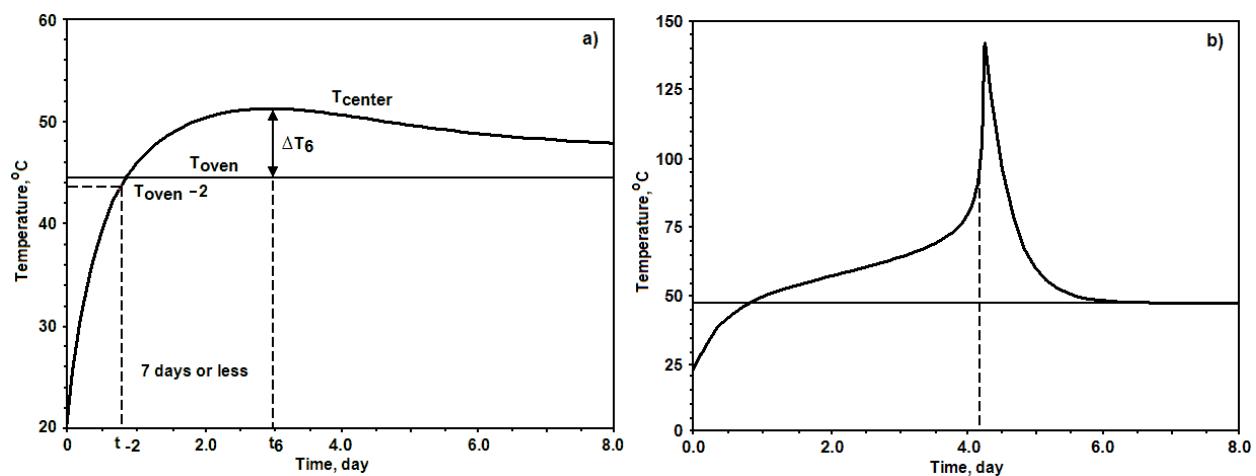
The characteristic 6-degrees overheating  $\Delta T_6$  may mean a conservative estimate of the critical temperature rise known from the thermal explosion theory (typical value of this parameter is 10 – 20 °C). The origin of the 7 day period is unintelligible. One can only guess that the 7-days period has been chosen assuming that longer transportation time is very unlikely; therefore a possibility of an explosion to occur after a lapse of this period is out of interest. Such an assumption is quite arguable because even 10 – 15 days transportation is not at all uncommon, not to mention about accidental delays.

Bearing in mind the abovementioned uncertainties it is important to understand in more detail how the SADT correlates with the critical temperature of thermal explosion  $T_{CR}$  which, for a packaging of given size, delimits the explosive and non-explosive domains of reaction proceeding and represents fundamental attribute of an explosion. To answer this question we considered two cases when the simple first-order reaction and the autocatalytic reaction occur in a product ( $\rho=1000 \text{ kg/m}^3$ ,  $c_p=2000 \text{ J/kg/K}$ ). In both the cases an explosion in the barrel of 0.6 m height and 0.2 m radius ( $S = 1 \text{ m}^2$ ,  $V = 75 \text{ l}$ ) had been simulated assuming that temperature distribution in the barrel is uniform (model of a well stirred tank, hereafter referred to as the lumped system). This model is suitable for low-viscous liquids. The initial temperature  $T_0$  is 20 °C, boundary conditions of the 3-rd kind with heat transfer coefficient  $U=4.7 \text{ W/m}^2/\text{K}$  were specified on all the external surfaces of the barrel. Mass of a product was 75 kg.

### Case 1. The first order reaction:

$$\frac{dQ}{dt} = Q^\infty k_0 e^{-\frac{E}{RT}} (1-\alpha) ; k_0 = 1.19 \times 10^9 \text{ s}^{-1}; E = 93.6 \text{ kJ/mol}; Q^\infty = 500 \text{ J/g} \quad (1)$$

The SADT (Fig. 1a) equals to 44.5 °C (Fig. 1a). The temperature course of the reaction reveals that it proceeds in the non-explosive domain.  $\Delta T_6$  is reached after a lapse of ~2.2 days.  $T_{CR}$  for the barrel (Fig. 1b) is 46.7 °C, the induction period is about 4 days.

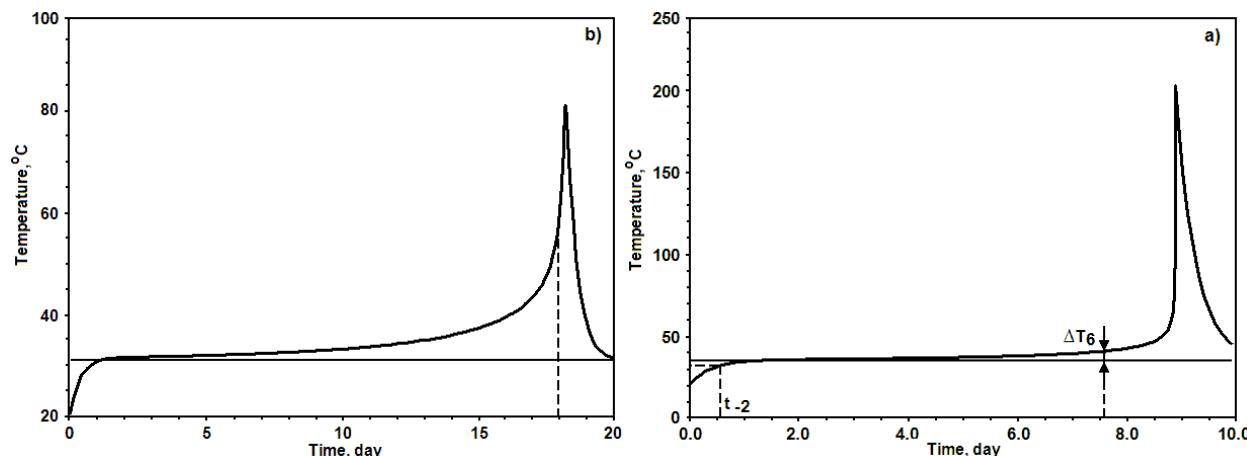


**Fig. 1. Determining SADT along the H1 test: the first-order reaction, lumped system.**  
**(a) – the ambient temperature equals SADT (H1 test);**  
**(b) - the ambient temperature equals the critical temperature of thermal explosion.**

### Case 2. The autocatalytic reaction:

$$\frac{dQ}{dt} = Q^\infty k_0 e^{-\frac{E}{RT}} (1 - \alpha)(z + \alpha); k_0 = 4.84 \times 10^9 \text{ s}^{-1}; E = 90 \text{ kJ/mol}; Q^\infty = 500 \text{ J/g}; z = 0.03 \quad (2)$$

Fig. 2 depicts the results of simulation. In this case the SADT equals to 34.8 °C (Fig. 2a),  $\Delta T_6$  is reached after a lapse of 7 days, and the explosion occurs soon after reaching  $\Delta T_6$ .  $T_{CR}$  (Fig. 2b) is 31.2 °C, the induction period is about 18 days. It is obvious that at the SADT determined in accordance with definition (D1) the reaction proceeds in the explosive domain far above the criticality.



**Fig. 2. Determining SADT along the H1 test: the autocatalytic reaction, lumped system.**

(a) – the ambient temperature equals SADT (H1 test);

(b) - the ambient temperature equals the critical temperature of thermal explosion .

Let us now determine the SADT for a solid substance when heat transfer in is governed by thermal conductivity (substance properties are the same as indicated above). In this case temperature distribution across the vessel is essential. The H1 test has been simulated for the same barrel by using the complete model with distributed parameters [11] (distributed system).

The results simulated are presented in Table 2 together with the results for the lumped system.

The non-uniformity of a system causes quite big difference in the SADT and  $T_{CR}$  for the first-order reaction. Diminution of thermal conductivity results in lowering of the SADT and  $T_{CR}$  so that the packaging with a solid product can even pass into the group 2 (Table 1) instead of 3. The SADTs and critical temperatures for the autocatalytic reaction are less sensitive to change of the heat transfer mechanism and variation of thermal conductivity.

Table 2: Comparison of SADT and  $T_{CR}$  for lumped and distributed systems

Type of the system	First-order reaction		Autocatalytic reaction	
	SADT, °C	$T_{CR}$ , °C	SADT, °C	$T_{CR}$ , °C
Lumped	44.5	46.7	34.8	31.2
Distributed, ( $\lambda=0.6$ W/m/K)	38.7	41.6	32.7	27.2
Distributed, ( $\lambda=0.1$ W/m/K)	28.5	31.4	28	20.9

Specific feature of the autocatalytic reaction explains this fact. Namely, the initial reaction rate is very low; reaction accelerates mostly because of accumulation of the product-catalyst. During the main part of the induction period heat is evolved slowly and its amount is rather small (see [11, 12] for more details). Therefore the system turns out to be closer to uniformity so that for solids with high and moderate thermal conductivity the lumped system model properly predicts the SADT. Note that because of small amount of heat which is accumulated in a substance during the induction period  $\Delta T_6$ , in contrast to the non-self accelerating reaction, is reached just before the explosion occurs.

These examples clearly demonstrate one intrinsic peculiarity of the SADT defined in accordance with (D1) – for non-self-accelerating reaction the SADT is always below  $T_{CR}$  whereas for autocatalytic reaction the SADT can be much higher than  $T_{CR}$ . The difference between the SADT and  $T_{CR}$  depends on the reaction kinetics, but the tendency remains in force. It can be shown that the same feature is valid for complex multi-stage reactions.

The observations discussed lead to several important conclusions:

1. Mechanism of heat transfer in a substance essentially affects critical temperature irrespective of the type of a reaction. The SATD is sensitive to mechanism of heat transfer; this effect ranges from quite strong for non-self-accelerating reactions to moderate for autocatalytic reactions.
2. The SADT defined in accordance with (D1) is reasonable indicator of criticality for non-autocatalytic reactions (though it can be somewhat conservative).
3. In case of autocatalytic reactions the SADT doesn't give any information about critical conditions but the SADT is essentially higher than critical temperature.

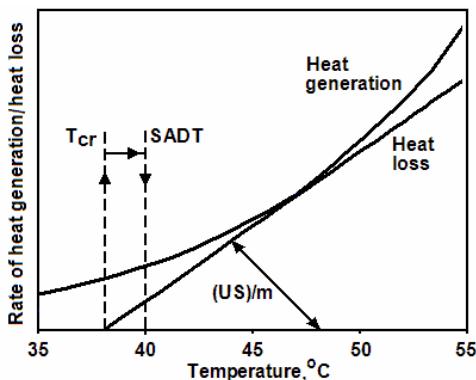
## 2.2 The Adiabatic and Isothermal Storage tests H2 and H3

The H2 and H3 tests are based on the different definition of the SADT:

**SADT is the critical ambient temperature rounded to the next higher multiple of 5 °C** (D2)

Both these tests are laboratory-scale experimental methods. The specific rate of heat generation evaluated from the corresponding calorimetric data is plotted on the Semenov

diagram (Fig. 3) together with the straight line of the specific heat loss for a commercial packaging.



**Fig. 3. Determining SADT in accordance with the H2 and H3 tests.**

Ambient temperature at which the heat loss line becomes the tangent to the heat generation curve represents critical temperature of thermal explosion.

This principle of the SADT determination implies that the H2 and H3 tests are essentially based on the lumped system model (the Semenov model of thermal explosion is valid only for a lumped system). Therefore the first limitation is that they cannot be applied for characterizing solid products.

The H2 and H3 tests differ from each other in calorimetric technique used for experimental investigation, and in the reaction types that can be assessed.

The H2 test exploits adiabatic calorimetry. The heat generation rate is evaluated from the self-heat rate data taking into account thermal inertia of the adiabatic bomb. The resultant data contain information about reactant consumption and temperature dependency of a reaction.

The H3 test is based on the use of isothermal calorimetry. Therefore series of experiments at different temperatures should be implemented to determine temperature dependency of a reaction rate. Moreover, in accordance with the test procedure the maximal rate of heat generation should be drawn on the Semenov diagram. It results in two important features:

1. In case of non-self acceleration reaction maximal rate occurs at the very beginning of a reaction. Therefore the heat generation rate curve on the Semenov diagram will not take into account the reactant consumption (as if the reaction were of zero-order) and  $T_{cr}$  evaluated from the diagram will be lower than the real critical temperature.
2. In case of autocatalytic reaction  $T_{cr}$  evaluated from the diagram will represent the correct critical temperature. As it was shown by Merzanov [12], author of the quasi-stationary theory of thermal explosion for autocatalytic reactions, the Semenov method can be applied for evaluating  $T_{cr}$  for such reactions provided that the maximal reaction rate is used instead of initial one.

This overview reveals additional limitations of the tests.

1. The H2 test cannot give reliable estimates if a reaction is autocatalytic. Moreover it is unusable for complex reactions because of the limitations of the Semenov theory.
2. The H3 test is capable of proper estimation of  $T_{CR}$  for autocatalytic reactions, but will always result in conservative estimates of  $T_{CR}$  for non-self acceleration reactions. Applicability of this test in case of complex reaction requires special analysis.

Let us now apply the H2 and H3 tests for determining the SADT for the same two cases from previous section. The results for the lumped system are presented in Table 3.

Table 3: Comparison of the SADTs calculated in accordance with H1, H2 and H3 tests

Test	First-order reaction		Autocatalytic reaction	
	SADT, °C	$T_{CR}$ , °C	SADT, °C	$T_{CR}$ , °C
H1	44.5	46.7	34.8	31.2
H2	45	44.8	40	37.5
H3	45	43.3	35	30.1

All the tests discussed give nearly the same SADT value for the first-order reaction. As it was predicted the Isothermal test H3 slightly underrates  $T_{CR}$ , but it doesn't affect the SADT estimate. In case of the autocatalytic reaction the H2 test results in the noticeably inflated values of the SADT and  $T_{CR}$ .

It should be emphasized that in case of the pronounced autocatalysis the difference in definitions of the SADT the tests H1 and H3 are based on (compare (D1) and (D2)) may result in serious inconsistency of the values. For instance, if  $T_{CR}$  determined by the H3 test for the autocatalytic reaction were just 0.2 degrees lower, i.e. 29.9 °C, then the SADT would be 30 °C which is by ~5 degrees lower than determined by using the H1 test. Let us cite another example related to the same barrel as discussed earlier, which contains organic peroxide. Its decomposition is highly exothermic (the overall heat effect is ~2000 J/g) and is characterized by strong autocatalysis. The SADT calculated according to the H1 test is 51 °C,  $T_{CR}=32.5$  °C. The H3 test gives precisely the same value of  $T_{CR}$  so that the SADT=35 °C. The H1 test suggests that for this peroxide assignment of control temperature is not required (the SADT >50 °C) whereas the H3 test results indicate that the product should be attributed to Group 2 (Table 1)!

### 2.3 The Heat Accumulation Storage test H4

The H4 test is based on the same SADT definition (D1) as the H1 test and the same procedure is used for determination. The main difference is that the small Dewar vessel (up to 1 liter) filled with the tested substance is used for experiments instead of a commercial packaging. Therefore some scale-up of the results on the full-size packaging is required. This is the key problem of the test.

Several scale-up methods have been proposed and are applied in practice. How to choose any certain method and which one is better? The answer strongly depends on the physical state of a product and size of a packaging under interest.

### 2.3.1 TDG scale-up procedure

The TDG suggests that the SADT determined by using the H4 test will be representative for a commercial packaging or IBS if the specific heat loss (in W/kg/K) is the same for the Dewar vessel and the packaging:

$$\left( \frac{US}{V} \right)_P = \left( \frac{US}{V} \right)_D , \quad (3)$$

where indices P and D denote packaging and Dewar respectively.

This condition is easily derived from the heat balance equation for the lumped system. The important and very useful practical feature of the scale-up condition (3) (and of the Semenov theory in general) is that it doesn't depend on the specific geometry of a vessel but only on the ratio of the surface of a vessel to its volume.

The TDG also suggests determining specific heat loss by measuring half-cooling time  $t_{1/2}$  for a packaging:

$$\frac{US}{V} = \rho c_p \frac{\ln 2}{t_{1/2}} \quad (4)$$

This scaling method is valid only for a well-stirred tank and, strictly speaking, the H4 test can be applied only for low-viscous liquids because in this case the temperature distribution in the Dewar vessel and in a packaging is approximately uniform.

Applicability of the H4 test for determining the SADT for solids, when internal heat transfer is governed by thermal conductivity, is perhaps the most disputable issue related to the SADT (see, for instance, recent publications [6-10]) because of the complexity of the scale-up problem. Therefore we will consider it in more detail.

Just as the scale-up method for liquids is based on the Semenov theory the scale-up for solids must be derived from the Frank-Kamenetskii theory (we deliberately consider only the simplest theories). Unfortunately there are several factors that hamper in direct application of this theory.

1. The theory had been created assuming that temperature on the surface of a solid body is defined (boundary conditions of the first kind). Contrary to it heat losses along the Newtonian law are typical for transportation or storage conditions (boundary conditions of the third kind).

2. This stationary theory doesn't consider development of a process in time whereas the SADT involves time (approximate of explosion induction period) as the essential parameter.
3. The theory gives analytical relations that are mostly applicable to the bodies of the simplest shapes – sphere, infinite cylinder and infinite slab. Many practical shapes such as barrel or box remain above its range.

### 2.3.2 Scale-up based on similarities between Semenov and Frank-Kamenetskii theories

For the first time the possibility to apply the results of the Semenov theory for approximate analysis of thermal explosion development in solid bodies of simple shapes was demonstrated by Frank-Kamenetskii [13]. Based on the formal similarity of the critical conditions for the lumped and the distributed system

$$\frac{E}{RT_0^2} Q k_0 e^{-E/RT_0} = \frac{1}{e} \frac{US}{V} \quad \text{lumped system}$$

$$\frac{E}{RT_0^2} Q k_0 e^{-E/RT_0} = \frac{\lambda}{r^2} \delta_{cr} \quad \text{distributed system} \quad (5)$$

Frank-Kamenetskii derived that the results of the Semenov theory can be approximately applied to solid bodies of simple shapes if to use the effective value of the heat transfer coefficient:

$$U_0 = \frac{V}{Sr^2} \lambda e \delta_{cr} \quad (6)$$

where  $r$  denotes the characteristic size (radius for a sphere or cylinder, half-thickness of an infinite slab).

Grewer [14] proposed to apply this idea for scaling-up the results of H4 test on the commercial packaging. Specifically he showed that the Dewar test performed for a self-reactive powder in a  $500 \text{ cm}^3$  Dewar flask with  $U_0 \approx 0.33 \text{ W/m}^2/\text{K}$  will be representative for a spherical packaging with  $r=0.27 \text{ m}$  calculated from (6) at  $\delta_{cr}=3.32$ , which corresponds to the volume of about 80 l (see also [8]).

Unfortunately there are several principal arguments against this scale-up method. As a matter of fact the very similarity between the critical conditions for the lumped and the distributed systems (5) is purely formal and doesn't have solid physical grounds. Nevertheless the concept of an effective heat transfer (6) can be used for rough estimates of explosion development in a solid but only under conditions of the first kind. It is quite evident from the expression (6), which doesn't contain real heat losses but characterizes only internal heat transfer governed by thermal conductivity. For instance, Grewer's results correspond to a packaging with Biot criterion  $Bi > 30$  which means that the H4 test from the example cited by Grewer is in fact representative for a packaging well under boundary conditions of the first kind.

The boundary problem of the explosion theory (an explosion under condition of Newtonian heat exchange with environment) had been considered in detail in [15]. Authors proposed more general approximate expression for effective value of the heat transfer coefficient  $U_{\text{eff}}$  that takes into account both internal heat transfer and external heat exchange  $U$ :

$$U_{\text{eff}} = \frac{U \cdot U_0}{U + U_0}, \quad U_0 = \frac{V}{S r^2} \lambda e \delta_{\text{cr}} \quad (7)$$

Bowes [16] showed that by substituting this effective coefficient in the condition (3) instead of the real value  $U$  one can achieve more reliable scaling-up of the H4 test results. Nevertheless this scale-up method is still applicable only to simple forms and, hence, doesn't allow correct estimation of the SADT for many practical cases. Moreover, it is principally inapplicable if a complex exothermic reaction proceeds in a product (including autocatalytic reactions) because neither Semenov nor Frank-Kamenetskii theory covers such cases.

### 2.3.3 Scale-up based on equality of half-cooling times – the HCT method

In the case of a solid substance specific heat loss doesn't have definite physical meaning therefore any attempts to estimate this parameter for a Dewar flask on the basis of the calibration of a packaging lead to wrong results. This makes it impossible to use the parameter for scaling-up. Nevertheless it turns out to be possible to approximately scale-up the results of the Dewar test if the same specific heat loss or, which is the same, half-cooling time as for a packaging can be provided for a Dewar flask experimentally. It will be demonstrated later that this method allows obtaining somewhat conservative estimate of the SADT. . Unfortunately in majority of practical cases the equality of half-cooling times cannot be ensured.

### 2.3.4 Scale-up based on the theory of regular cooling mode

One can propose more universal scale-up method based on providing thermal equivalence of solid bodies of different size and even of different shapes having different physical properties. The theoretical ground of the method is the concept of regular cooling mode introduced by Kondratiev [17].

Let us consider temperature variation in inert solid bodies of simple shapes (sphere, slab, infinite and finite cylinder, parallelepiped) heated in an environment with constant temperature  $T_e$  (boundary conditions of the third kind). Temperature in any point of a body is represented by the infinite series [18].

$$\frac{T - T_e}{T_0 - T_e} = \sum_{n=1}^{\infty} \prod_{i=1}^3 A_{n,i} X_{n,i} \exp\left(-\frac{\mu_{n,i}^2}{r_i^2} at\right). \quad (8)$$

Here  $A_{n,i}$  stand for initial thermal amplitudes that depend on initial temperature distribution and body shape,  $X_{n,i}$  are geometry-dependent functions,  $r_i$  denote characteristic dimensions of a body;  $\mu_{n,i}$  are the roots of the characteristic equations, they are complex tabular functions of  $Bi$ :  $\mu_{n,i} = \mu_{n,i}(Bi_i)$ ,  $Bi_i = Ur_i/\lambda$ .

After a lapse of the transient period  $\tau_t$  only the first term of the series (8) remains significant and the regular mode of cooling is set in:

$$\frac{T - T_e}{T_0 - T_e} = \prod_{i=1}^3 A_{1,i} X_{1,i} \exp\left(-\frac{\mu_{1,i}^2}{r_i^2} at\right). \quad (9)$$

or, in the differential form

$$\frac{\partial(T - T_e)}{\partial\tau} = -\omega(T - T_e), \quad \frac{\partial \ln(T - T_e)}{\partial\tau} = -\omega; \quad \omega = a \sum_{i=1}^3 \frac{\mu_{1,i}^2}{r_i^2} \quad (10)$$

where  $\omega$  is the cooling tempo,  $\mu_{1,i}$  -the first roots of the corresponding characteristic equations.

The regular cooling (or heating) mode is distinguished by several important features.

1. At the expiration of the transient period the logarithmic rate of temperature variation in any point of a solid body of any shape regardless of the initial temperature distribution becomes identical and constant.
2. The cooling tempo  $\omega$  depends on heat transfer coefficient (through  $\mu_1$  (Bi)) and on thermal diffusivity of a substance. Thus  $\omega$  represents an integral characteristic that gives proper weigh of external heat exchange and internal conductive heat transfer within a solid substance.
3. Matching the cooling tempos for vessels of different shape and size having different physical properties ensures equivalence of their thermal behavior. Specifically, a Dewar flask and a commercial packaging will be equivalent if

$$\omega_D = \omega_P. \quad (11)$$

Strictly speaking this condition of thermal equivalence is valid only for inert systems. For a self-reacting substance only approximate equivalence can be observed provided that heat generation due to an exothermic reaction is small and deviation of a reactive system from the inert one is also small. Usually this requirement is fulfilled during the most part of the induction period especially in the vicinity of criticality. In particular this is the case when the SADT is to be determined because the overheating doesn't exceed 6 °C.

The scale-up method based on regular cooling mode (hereafter referred to as the RCM method) has several essential advantages.

1. The cooling tempo can be easily calculated from (10) for bodies of different shapes if thermal-physical properties of a substance and external heat transfer coefficient are known.

For simple shapes (sphere, infinite cylinder and infinite slab) (10) is reduced to the formula

$$\omega = a[\mu_1(Bi)/r]^2 \quad (12a)$$

where  $r$  is the characteristic dimension (radius for a sphere or cylinder and half-thickness for a slab); the function  $\mu_1(Bi)$  in tabular form can be found in [18, 19] (see also Appendix A).

As it follows from (10) cooling tempos for bodies of more complex shapes are calculated on the basis of the superposition principle. Thus, a finite cylinder (barrel) can be interpreted as the intersection of an infinite cylinder and slab, therefore

$$\omega = a[\mu_{1s}^2 / (h/2)^2 + \mu_{1c}^2 / r^2]; \quad Bi_s = \frac{U_s h / 2}{\lambda}; \quad Bi_c = \frac{U_c r}{\lambda}, \quad (12b)$$

where indices  $s$  and  $c$  denote slab and cylinder respectively;  $\mu_{1s}$  and  $\mu_{1c}$  represent the first roots of the characteristic equations for infinite slab and infinite cylinder;  $r$  is radius of a cylinder,  $h$  is its height.

A parallelepiped is the intersection of three infinite slabs, therefore

$$\omega = a \sum_{i=1}^3 \left( \frac{\mu_{1s}(Bi_{si})}{h_i/2} \right)^2; \quad Bi_{si} = \frac{U_{si} h_i / 2}{\lambda}, \quad (12c)$$

where  $h_1$ ,  $h_2$  and  $h_3$  represent dimensions of a parallelepiped.

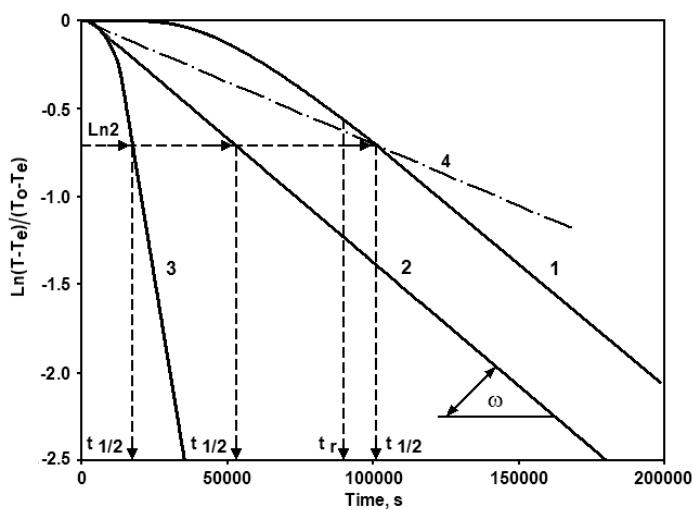
2. The cooling tempo can be determined experimentally by using an inert solid substance or a reactive substance at temperatures where a reaction is negligibly slow.

Fig. 4 depicts typical cooling curves for spherical vessels of different size with a solid substance ( $c_p = 2000 \text{ J/kg/K}$ ,  $\rho = 1000 \text{ kg/m}^3$ ,  $\lambda = 0.2 \text{ W/m/K}$ ). Curves 1 and 2 represent cooling of the thermally equivalent vessels of significantly different size, the equality of  $\omega$  is provided by selecting the appropriate values of heat transfer coefficient ( $U = 10 \text{ W/m}^2/\text{K}$  for the large vessel as against  $U = 0.456 \text{ W/m}^2/\text{K}$  for the small one). Curve 3 demonstrates significant increase of  $\omega$  for a vessel of a medium size with the same specific heat transfer  $US/V$  as for the large one (in both the cases  $US/V = 120 \text{ W/m}^3/\text{K}$  so that for the medium vessel  $U = 6 \text{ W/m}^2/\text{K}$ ). Note that for the small vessel, which is thermally equivalent to the large vessel  $US/V = 27.4 \text{ W/m}^3/\text{K}$ .

Fig. 4 vividly illustrates the complete inapplicability of the concept of the specific heat transfer to solids emphasized by Fierz [6]. In case of a packaging with a solid both internal heat transfer governed by thermal conductivity and external heat losses from the surface are of key importance. Contributions of these mechanisms

essentially depend on thermal diffusivity of a substance, heat transfer coefficient, and geometry and dimensions of a package. From this point of view results of packaging calibration cannot be transferred on the same packaging containing any other solid substance with different physical properties. It is in contrast with TDG recommendation to use dicyclohexyl phthalate as a calibration substance.

Furthermore, transient period that precedes the regular mode goes up significantly with increase of a packaging size and can be comparable or even longer than half-cooling time (compare curves 1 and 2 in Fig4).



**Fig.4 Cooling of spherical vessels**

1 –  $r=25 \text{ cm}$ ; 2 –  $r=5 \text{ cm}$ ; 3 –  $r=15 \text{ cm}$ ; 4 – a small vessel has the same  $t_{1/2}$  as a large one;

$T_0=80^\circ\text{C}$ ,  $T_e=20^\circ\text{C}$ ,  $U_1=U_3=10 \text{ W/m}^2/\text{K}$ ;  $U_2=0.456 \text{ W/m}^2/\text{K}$ .

It can lead to some confusing results. Thus, the small vessel would have the same half-cooling time as the large one (curve 4, Fig.4) if  $U$  were  $0.24 \text{ W/m}^2/\text{K}$ , i.e.  $U$  would be almost two times smaller than it is required for thermal equivalence. It demonstrates once more that the half-cooling time cannot be used as a proper indicator of thermal equivalence.

3. The fact that the cooling tempo measured experimentally has well defined physical meaning allows applying various ways of a vessel calibration.

A cooling experiment can be performed by using some inert solid substance with physical properties different form those of a reacting product. Then  $U$  is calculated by using one of the formulas (12a)-(12c) with the following calculation of  $\omega$  for the solid product under interest.

In the same way the results of calibration of a commercial packaging allow calculation of  $U_D$  for a Dewar flask that will ensure thermal equivalence and vice versa.

The following examples illustrate these possibilities and will allow several useful conclusions.

Example 1: The cooling tempo  $\omega=2.67 \cdot 10^{-5} \text{ s}^{-1}$  has been determined (simulated in our case) for the commercial spherical packaging ( $r=0.25 \text{ m}$ ) containing a solid product ( $c_p=1000 \text{ J/kg/K}$ ,  $\rho=1000 \text{ kg/m}^3$ ,  $\lambda=0.2 \text{ W/m/K}$ ,  $a=2 \cdot 10^{-7} \text{ m}^2/\text{s}$ ).

a. In accordance with (12a)  $\mu_{1,P} = r_P \sqrt{\omega/a} = 0.25 \sqrt{2.67 \cdot 10^{-5} / 2 \cdot 10^{-7}} = 2.89$ .

This value corresponds to  $Bip=U \cdot r_p / \lambda = 12.5$  (Table A1 of Appendix A) hence  $U_p = Bip \cdot \lambda / r_p = 12.5 \cdot 0.2 / 0.25 = 10 \text{ W/m}^2/\text{K}$  for the packaging.

b. Now we can calculate  $U_D$  for the spherical Dewar with  $r_D=0.05 \text{ m}$  filled with the same substance. The cooling tempo for the Dewar must be the same as for the packaging, therefore  $\mu_{1,D} = r_D \sqrt{\omega/a} = 0.05 \sqrt{2.67 \cdot 10^{-5} / 2 \cdot 10^{-7}} = 0.578$ .

From Table A1 we get  $Bi_D=0.113$  so that  $U_D = Bi_D \cdot \lambda / r_D = 0.113 \cdot 0.2 / 0.05 = 0.452 \text{ W/m}^2/\text{K}$ . The SADTs determined for the Dewar flask and the packaging are presented in Table 4. It demonstrates also the SADTs determined for the same spherical Dewar flask on the basis of other scale-up methods. The SADT values determined by using the US H1 test (also simulated in our case) are considered here as the references.

Table 4 : The SADTs for thermally equivalent spherical Dewar and packaging

Reaction type	US test H1 (packaging)	Heat accumulation storage test H4, different scale-up methods			
		RCM ( $\omega=\text{const}$ )	$t_{1/2}=\text{const}$	Bowes ( $U_{\text{eff}}S/V=\text{const}$ )	TDG ( $US/V=\text{const}$ )
		$U_D=0.452^*$	$U_D=0.24^*$	$U_D=0.4^*$	$U_D=2^*$
N-order	33.6	35.9	30.3	34.5	48.4
Autocatalytic	30.3	31.7	29.8	31.3	36.9

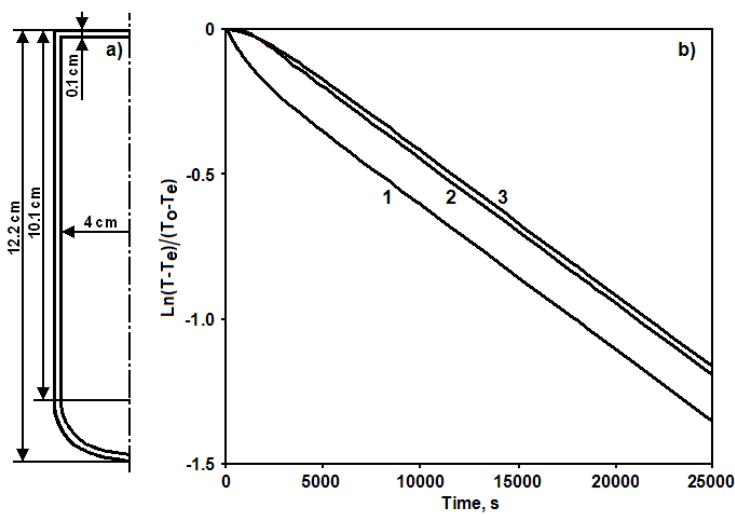
\*) in  $\text{W/m}^2/\text{K}$

Example 2 represents more complex case of a real Dewar flask with round bottom described in [10] (see Fig. 5a) with different heat losses on different surfaces, specifically  $U_{\text{top}}=3.5 \text{ W/m}^2/\text{K}$  whereas  $U_{\text{side}}=U_{\text{bottom}}=0.29 \text{ W/m}^2/\text{K}$ . The flask is filled with a solid reactive substance ( $\rho=464 \text{ kg/m}^3$ ,  $\lambda=0.16 \text{ W/m/K}$ ,  $c_p=1450 \text{ J/kg/K}$ ). The inner flask is supposed to be made of stainless steel: wall thickness is 1 mm,  $\rho=7000 \text{ kg/m}^3$ ,  $\lambda=16 \text{ W/m/K}$ ,  $c_p=500 \text{ J/kg/K}$ . The effective thermal inertia of the Dewar is 1.37, i.e. the same as indicated in [10].

- a. Thermal behavior of this object has been simulated numerically. The pronounced temperature distribution along the symmetry axis appears in the course of cooling. Fig. 5b depicts variation of relative temperature in three different points – near the

top (curve 1), in the middle (curve 2) and close to the bottom (curve 3). After a lapse of the transient period the regular cooling mode is set and the cooling tempo in all the points becomes the same:  $\omega=4.97 \cdot 10^{-5} \text{ s}^{-1}$ .

- b. Now one can calculate the parameters of a barrel (commercial packaging) that will be represented by this Dewar flask. Under the assumption that  $h_p=2r_p$  and that heat losses are the same on all the surfaces formula (13b) gives:  $\omega=a(\mu_{ls}^2 + \mu_{lc}^2)/r^2$ ;  $Bi_s = Bi_c = U_p * r / \lambda$ . The packaging contains the same solid substance so that  $a=2.38 \cdot 10^{-7} \text{ m}^2/\text{s}$ . There are two unknown parameters – radius  $r_p$  and the heat transfer coefficient  $U_p$  therefore one can estimate  $U_p$  for a barrel of given size or calculate the size for the given  $U_p$ .



**Fig. 5. Cooling of the Dewar flask.**

a) – Sketch of the inner flask; b) - Temperature variation in different locations:  
1 – 1 cm from the top, 2 – the middle, 3 - 1 cm from the bottom.

1. The barrel size is assigned:  $r_p = 0.18 \text{ m}$  (volume of the barrel is  $\sim 36 \text{ l}$ ). The appropriate values of the first roots should be found in Table A2 and A3 (Appendix A). They must correspond to the same value of  $Bi$  and the sum of their squares must be equal to  $\omega r^2 / a = 6.76$ . The sought for values correspond to  $Bi=9.71$ :  $\mu_{ls}=1.43$  and  $\mu_{lc}=2.17$ . Finally the required value of  $U_p$  is  $8.6 \text{ W/m}^2/\text{K}$ . It should be emphasized that the correlation between packaging size and intensity of heat exchange is very strong. Thus, for a barrel of radius  $r_p = 0.2 \text{ m}$  (volume is  $\sim 50 \text{ l}$ )  $Bi$  will be more than 100, i.e. the barrel proves to be under the conditions of the first kind.
2. It is known that  $U_p = 3 \text{ W/m}^2/\text{K}$ . In this case determination of the appropriate values of  $\mu_{ls}$  and  $\mu_{lc}$  requires several iterations: First,  $Bi$  is calculated for some initial

guess on  $r$ , then values of  $\mu_{1s}$  and  $\mu_{1c}$  are evaluated, cooling tempo  $\omega_p$  is calculated and compared with  $\omega_D$ . If  $\omega_p \neq \omega_D$  the next iteration is implemented with the changed value of  $r$ . In our case the resultant values of the first roots are  $\mu_{1s} = 1.16$ ,  $\mu_{1c} = 1.74$ , and radius of the barrel is 0.145 m ( $Bi=2.72$ ).

Table 5 represents the calculated SADTs for the Dewar flask and the equivalent barrels.

Table 5 : The SADTs for thermally equivalent Dewar and barrels

Vessel	SADT, °C	
	First-order reaction	Autocatalytic reaction
Shelled Dewar, $r=0.04$ m	47	36.5
Barrel, $r=0.18$ m ( $U=8.6$ W/m <sup>2</sup> /K)	42	34.4
Barrel, $r=0.145$ m ( $U=3$ W/m <sup>2</sup> /K)	42.7	34.7

As in the previous cases the resultant SADTs for the autocatalytic reaction are close enough to each other and no significant sensitivity to the approximate nature of the scale-up method is observed. Results for N-order reaction are more sensitive so that difference between the SADTs for the Dewar and the barrels reaches about 5 °C.

### 2.3.5 Comparative analysis of Scale-up methods

General discussion of capabilities and limitations of four different scale-up methods have been presented in previous sections. The results presented in Table 4 allow implementation of comparative analysis of these methods.

We could see that three different scale-up methods resulted in obtaining comparable estimates.

The **RCM scale-up method** ensures reasonable correspondence between the SADTs determined though the H4 test gives somewhat overstated values. In case of an autocatalytic reaction the results are less sensitive to the approximate nature of scaling. Limitation of method is that cooling tempo cannot be calculated analytically for such complex objects as the real Dewar (complex geometry or different and asymmetrical heat losses on different surfaces). In these cases the RCM method allows only one-way scale-up – from a Dewar to a packaging, therefore the cooling tempo for a Dewar should be determined experimentally. The use of numerical simulation allows applying the RCM method in full measure to complex systems.

Estimates provided by the **Bowes method** are in good accord with results of the RCM method. Nevertheless the Bowes method has several serious limitations. Two of them have been mentioned already – inapplicability in case of complex geometry or reaction mechanism. Another limitation is that it cannot be applied if heat losses are asymmetrical. Moreover, detailed knowledge about thermal properties of a system is required, in particular heat transfer coefficient

should be known for a packaging or a Dewar. As it was mentioned, this parameter can be reliably evaluated only from cooling tempo after its determination, i.e. Bowes method should be applied in combination with the RCM –based calibration.

Scale-up based on **equality of half-cooling times** also allows obtaining reasonable estimates but on a conservative side. The origin of this conservatism has been discussed earlier (see Fig.4). Important note is that this scale-up method will give such results only provided that half-cooling times were determined by direct measurement for every specific substance, Dewar and packaging. Important problem is that, because this method is purely empirical when it concerns solids, it is impossible to predict how variation of geometry, physical properties and features of a reaction can affect reliability of the results.

Finally, the **TDG method** demonstrates total inadequacy.

Summarizing all these facts one can conclude that the RCM method, having better justified physical basis, has better prospects, especially in combination with methods of mathematical simulation.

Important practical observation is that in all the cases irrespective of the scale-up method applied it appeared that the ~400 ml Dewar flask, depending on its geometry and geometry of a package, can be reasonably representative for a packaging with the volume of about 30 - 40 l. Determination of the SATD for solid-containing packages of larger volume by using the H4 test is impossible.

The last note of this section concerns the reason why the results of the SATD determination for many organic peroxides by using the H4 and H1 tests are in good agreement (see statement in [8- 10]). We believe that the main reason is in autocatalytic decomposition mechanism which is typical for organic peroxides. We could see that in this case due to specific feature of self-accelerating reactions different methods for the SATD determination give very comparable results. It is also very likely that more correct half-cooling time scale-up method has been used, i.e .  $t_{1/2}$  was measured for the Dewar and a packaging.

### 3. Applying the kinetics-based simulation for SATD determination

Overview of the tests recommended by the TDG for determining the SATD reveal numerous problems that can be met while applying one or another method.

The H1 test is very time and cost consuming and cannot be used for big packages or tanks.

The H2 an H3 tests are rather flexible and cost-effective but they are principally inapplicable for solid products. In addition they are based on different definition of the SATD, which can lead to getting the results that are incomparable with the results of other methods.

The H4 test is also time consuming and is fraught with various problems when it concerns investigation of solid substances. Moreover this test cannot predict properly the SATDs for big packages or tanks.

There are several practical problems that are totally out of the scope of the TDG recommendations, namely:

- Determining the SADT for large-tonnage tanks (tank-trucks, tank-wagons), stacks of packages;
- Evaluating safety margins at transport of bulk cargos of self-reactive products (an example is transportation of ammonium nitrate-based fertilizers);
- Assessing potential hazards at transportation or storage of self-reactive products for more than 7 days.

In all these cases experimental methods for the SADT determination are either inapplicable at all or are very troublesome or problematic. Only one method is capable of responding to this challenge. This is the kinetics-based simulation approach which can be very beneficial addition to the tests. It comprises three main steps:

- implementing necessary series of calorimetric experiments;
- creating the mathematical model of a reaction on the basis of experimental data;
- incorporating the kinetic model into the model of a process and achieving the practical target by using mathematical (numerical) simulation.

The detailed discussion of the approach is out of the scope of this paper; it can be found in [20]. Here we will mention only that it can help in resolving various problems. Let us mention only two of them.

(1) The SADT can be determined (calculated) for a package, vessel of any size having various shapes.

(2) We could see that there are two different definitions of the SADT that may lead to inconsistent estimates. We also discussed the fact that the very concept of SADT contains some vagueness. As simulation allows calculation of any parameter or quantity with regard to thermal explosion development one can propose more precise unified definition of the SADT:

**SADT is the lowest environment (oven) temperature at which thermal explosion occurs in the specific commercial packaging after a lapse of the preset transportation time, otherwise SADT is the critical ambient temperature rounded to the next lower multiple of 5 °C.**

We deliberately avoided mentioning any fixed transportation time in the definition. It may be the same 7 days as in the TDG definition or any other value depending on the specific scenario.

The definition envisages two cases we met when analyzing the impact of a reaction mechanism on the SADT:

- For those cases when SADT is higher than critical temperature (autocatalytic reactions) the definition requires that induction period must exceed transportation time.
- It may happen that the induction period is less than transportation time even at critical temperature (this is the case for N-order reactions). Then the SADT is taken as critical ambient temperature rounded to the next lower multiple of 5 °C. In such a way the system is being located in the non-explosive domain thus ensuring safety at transportation.

Material presented in previous sections clearly demonstrates efficiency and usefulness of the kinetics-based simulation method for analysis of various scenarios – all the illustrative results were simulated. Let us present one more example.

This example demonstrates how the simulation method helps in solving the problem when neither of experimental methods is applicable. It concerns determination of the SADT for the stack of boxes. In accordance with TDG the SADT should be determined for a commercial package subject to transport. However it is usual practice to transport packaged goods in stacks rather than to carry every single packaging separately. Doubtless the SADT for a stack will differ from those for a single package. It is also obvious that this parameter cannot be estimated by either of the experimental tests.

Let us compare the shelled box of 20x20x20 cm size containing 7.5 kg of reactive solid product and the stack of 27 (3x3x3) boxes. The product decomposes along the single-stage first order reaction with  $E=110 \text{ kJ/mol}$ ,  $k_0=1.96 \cdot 10^{11} \text{ s}^{-1}$ ,  $Q^\infty=500 \text{ J/g}$ .

The container wall thickness is 2 mm. We will consider two cases – metallic container ( $\lambda=16 \text{ W/m/K}$ ) and container made of polymer ( $\lambda=0.2 \text{ W/m/K}$ ). In both these cases thermal conductivity of a product was the same:  $\lambda=0.15 \text{ W/m/K}$ .

Table 6 : SADT and critical temperature for a box and for a stack of boxes.

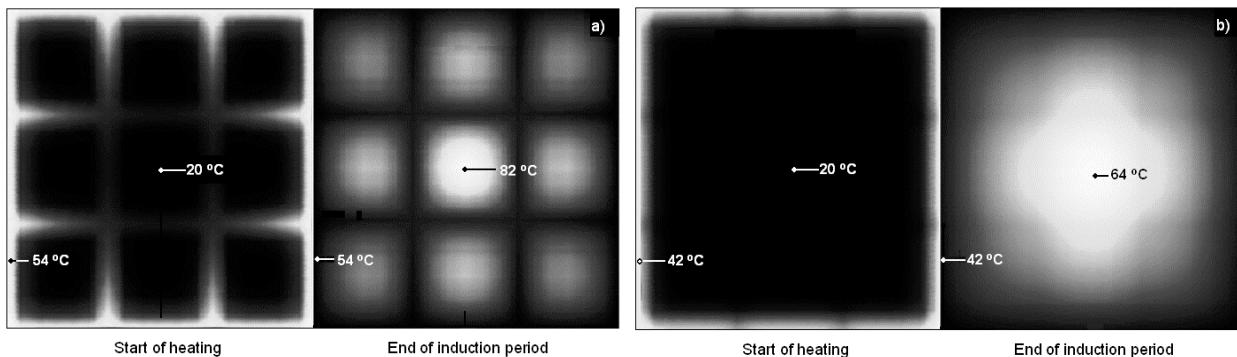
Container	Single box		Stack of boxes	
	SADT, °C	T <sub>CR</sub> , °C	SADT, °C	T <sub>CR</sub> , °C
Metallic	57	61	48	54
Polymer	55	57	39	42

The simulated results (Table 6) demonstrate that the SADT for the single box is much higher than those for the stack and even exceeds its critical temperature so that the use of this SADT for the stack will be absolutely unsafe. Thus, for boxes with polymer containers, the induction period of the stack explosion at ambient temperature 55 °C is 4.5 days, i.e. smaller than the permissible 7 days.

The remarkable detail of the results obtained is that significant difference between the SATDs and critical temperatures for the stacked-up boxes with metallic and polymer containers

is observed whereas single boxes with different container materials behave similarly. Analysis of the temperature fields in the stacks (Fig.6) gives the detailed explanation.

If containers are of metal the walls, though thin enough, serve as very efficient heat conducting elements. At the heating stage they facilitate external heat to penetrate into a stack (Fig. 6a, left drawing) thus accelerating the heating. On the contrary, when reaction heat release becomes significant metallic walls help to withdraw heat from a stack outwards (Fig. 6a, right drawing), which leads to elevation of the SADT and critical temperature.



**Fig. 6. Temperature distribution in the stack cross-section .**

**a) stack of boxes with metallic containers;**

**b) stack of boxes with polymer containers**

The polymer has about the same thermal conductivity as the reactant therefore the stack behaves almost as a monolithic box of the reactant of the same size as the stack so that both the SADT and critical temperature are much lower than for the single small box (Fig. 6b).

In spite of great capabilities of numerical simulation there are some serious difficulties that impede the wide use of this method. First of all, mathematical complexity of the approach renders it impossible to apply simulation without powerful computers and appropriate software. Moreover, detailed enough knowledge about reaction kinetics is required.

As far as software is concerned, there are several well-known commercial codes of general designation that can be used for thermal explosion simulation. Let us mention as the examples the CFX (ANSYS Inc.) and FLUENT (FLUENT Inc.) software. However, these universal codes are expensive, difficult for use and often are not efficient enough for solving specific problems. Therefore, development of problem-oriented software (an example is AKTS-TA-Software, Switzerland) is a challenging decision.

The Thermal Safety Software (TSS) series developed by Cheminform Ltd. (CISP), Russia is another example of the specialized software. This series comprises the ThermEx and ConvEx program packages designed for comprehensive simulation of thermal explosions in solids and liquids, set of programs for reaction kinetics evaluation, and some other applications. Therefore TSS supports all the stages of an investigation aimed at assessment of safety at transport of self-reactive products. Specifically, all the results presented in the paper were obtained by using the Fork program for simulation of well stirred tanks and the ThermEx package - for simulation of

processes in solids (distributed systems) and automatic SADT determination in compliance with the US SADT test.

## Conclusions

1. The principal limitation of the adiabatic and isothermal storage tests H2 and H3 is that they are unfit for determination of the SADT for solid products. Furthermore, the H2 test is fraught with obtaining erroneous and, more important, unsafe estimates of the SADT if an autocatalytic reaction proceeds in a product. Therefore it shouldn't be applied in such cases.
2. The H2 and H3 tests are based on different definition of the SADT as against the H1 and H4 tests. This difference can result in obtaining inconsistent estimates of the SADT for the same package. To avoid such kind of misleading it is highly desirable to use one definition for any of methods intended for the SADT assessment. As long as no new universal definition is proposed one can recommend to apply the definition based on permissible overheat (the H1 and H4 tests) which ensures reasonable estimates both for non-self accelerating and autocatalytic reactions.
3. The heat accumulation storage (Dewar) H4 test can be considered as restrictedly applicable for determining the SADT for solid products provided that the adequate scale-up method has been selected. In particular, the 400 ml Dewar flask can be representative for packages of up to approximately 30 liters. It was shown that the half-cooling time method can give reasonable results if this time interval is measured directly both for the flask and for a packaging. The Bowes method can be applied for simple determinations (simple geometry and kinetics).
4. The RCM method proposed in the paper represents the advanced, more universal approach to scaling though it requires use of mathematical simulation in complex cases. It was also demonstrated that this method provides the most correct determination of thermal physical properties of a vessel. Nevertheless it should be emphasized that all the scale-up methods are approximate ones and don't guaranty real thermal equivalence of reactive systems of different sizes and geometries.
5. Kinetics-based simulation approach is the general method for SADT determination. In some cases (complex geometry, complex reactions, SADT for stack of packages or bulked cargos, etc.) numerical simulation is the only way to get answers. Therefore it can be proposed as very useful and promising additional method.

All the demonstrated results have been obtained by using the software developed by Cheminform St.-Petersburg, Ltd. Specifically the Fork program was used for simulation of lumped systems and the ThermEx package - for simulation of processes in solids (distributed systems) and automatic SADT determination in compliance with the US SADT test.

## Nomenclature

E - activation energy, kJ/mol

$k_0$  – preexponential factor, s<sup>-1</sup>

$\alpha$  - degree of conversion

$Q^\infty$  – heat effect of a reaction, J/kg

dQ/dt – specific rate of heat generation due to a reaction, W/kg

$z$  - autocatalytic constant

R - universal gas constant, R= 8.31 J/mol/K

T – temperature, K

$T_o$  – initial temperature of a product, K

$T_e$  – ambient temperature, K

$T_{CR}$  – critical temperature of thermal explosion, K

$c_p$  – specific heat of a product, J/kg/K

$\rho$  – product density, kg/m<sup>3</sup>

$\lambda$  – thermal conductivity coefficient, W/m/K

a – thermal diffusivity,  $a = \lambda/c_p/\rho$ , m<sup>2</sup>/s

$\mu_i$  – roots of the characteristic equation

$\omega$  - cooling tempo, s<sup>-1</sup>

U – heat transfer coefficient, W/m<sup>2</sup>/K

S – surface of heat exchange, m<sup>2</sup>

V – volume of a vessel or a package, m<sup>3</sup>

h – height of a barrel, m

r – radius of a barrel, m

$m_p$  – mass of a product, kg

$\Delta T_6$  – the characteristic 6-degree overheat in the middle of a package,  $\Delta T_6 = 6$  °C

$T_{CR}$  – critical temperature of thermal explosion, K

(US)/m –specific heat loss, W/kg/K

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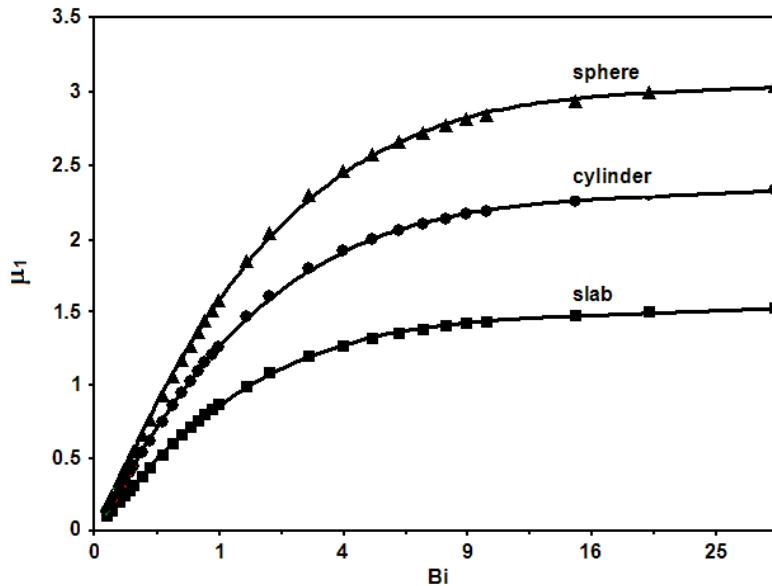
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### Appendix A – first roots of the characteristic equations

Bi	$\mu_1$		
	Sphere (A1)	Slab (A2)	Cylinder (A3)
$\operatorname{tg}\mu = -\mu / (\operatorname{Bi} - 1)$			$J_0(\mu) / J_1(\mu) = \mu / (\operatorname{Bi} - 1)$
0.02	0.24450	0.141	0.1995
0.04	0.34500	0.1987	0.2814
0.06	0.42170	0.2425	0.3438
0.08	0.48600	0.2791	0.396
0.1	0.54230	0.3111	0.4417
0.2	0.75930	0.4328	0.617
0.4	1.05280	0.5932	0.8516
0.6	1.26140	0.7051	1.0184
0.8	1.43200	0.791	1.149
1	1.57080	0.8603	1.2558
1.5	1.83660	0.9882	1.4569
2	2.02880	1.0769	1.5994
3	2.28890	1.1925	1.7887
4	2.45570	1.2646	1.9081
5	2.57040	1.3138	1.9898
6	2.65370	1.3496	2.049
7	2.71650	1.3766	2.0937
8	2.76540	1.3978	2.1286
9	2.80440	1.4149	2.1566
10	2.83630	1.4289	2.1795
15	2.93200	1.4729	2.2509
20	2.98750	1.4961	2.288
30	3.03700	1.5202	2.3261
50	3.07880	1.54	2.3572
Polynomial approximation of the dependency $\mu_1(\operatorname{Bi})$ : $\mu_1 = a_0 + \sum_1^4 a_i X^i$ ; $X = \sqrt{\operatorname{Bi}}$			
(13)			
$a_0$	- 0.0736	- 0.0288	- 0.0529
$a_1$	2.1513	1.2059	1.7423
$a_2$	0.5748	- 0.3724	- 0.501
$a_3$	0.0689	0.0518	0.0651
$a_4$	-0.0031	-0.0027	-0.0031
$R^2$	0.9997	0.9998	0.9998

The polynomial (13) provides sufficient precision of approximation (correlation coefficients for all the geometries are very close to 1) and can simplify calculations required when using the RCM method. Fig. 7 illustrates dependency of  $\mu_1(B_i)$  and quality of polynomial approximation.



**Fig. 7. Dependency of the first roots on  $B_i$ .**

■ ■ ■, ● ● ●, ▲ ▲ ▲ - tabular values; — - polynomial approximation